Transient population and polarization gratings induced by (1+1)-dimensional ultrashort dipole soliton

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An ultrafast transient population grating induced by a (1+1)-dimensional, ultrashort dipole soliton is demonstrated by solving the full-wave Maxwell-Bloch equations. The number of lines and the period of the grating can be controlled by the beam waist and the area of the pulse. Of interest is that a polarization grating is produced. A coherent control scheme based on these phenomena can be contemplated as ultrafast transient grating techniques.

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I. INTRODUCTION

Population gratings (PGs) have attracted much interest owing to their several possibilities of applications, such as laser diffraction on electromagnetically induced grating [1,2], self-cooling effect [3], self-starting laser oscillator [4], transient Bragg diffraction [5], nearly degenerate four-wave mixing in cold atoms [6], and high-order susceptibilities in cold atoms [7]. Recently, a population grating technique has been reported to characterize quantum dots [8]. In addition, Brown *et al.* have developed femtosecond transient grating techniques for the study of the molecular dynamics [9]. Scholes and co-workers have measured the exciton spin relaxation in quantum dots using ultrafast transient polarization grating spectroscopy [10]. However, common to the above works is that the standing wave is required.

With the development of the laser pulse technology, the light-matter interaction described by the one-dimensional Maxwell-Bloch equations is well discussed [11,12]. When very narrow optical beams propagate without affecting the properties of a medium, they undergo natural diffraction and broaden with distance. The narrower the initial beam is, the faster it diverges [13]. So the transverse effects play a critical role in the response of the system. Based on the coupled Maxwell-Bloch equations with the slowly varying envelope approximation (SVEA) and rotating-wave approximation (RWA), the influences of the transverse effects on the self-focusing of coherent optical pulses are investigated [14–17]. McCall *et al.* have revealed the dependence of the pulse area and energy on the transverse distribution of the field [18]. However, a number of works have reported the limitations of the SVEA [19–25]. Several new features arise in the full Maxwell solution that are absent in the standard SVEA models. Slavcheva *et al.* have investigated the two-dimension TM and TE guided modes in the parallel-plate mirror optical waveguide on the basis of the full-wave Maxwell-Bloch equations [26]. Forysiak and co-workers have reported the nonlinear focusing of femtosecond pulses as a result of self-reflection from a saturable absorber [27].

Here, we investigate the nonlinear response of a two-level medium driven by an ultrashort dipole soliton in the (1+1)-dimensional geometry. By solving the two-dimensional full-wave Maxwell-Bloch equations without SVEA and RWA, we demonstrate, for the first time to our knowledge, an ultrafast transient population grating induced by a (1+1)-dimensional, TE-polarized, ultrashort dipole soliton even if the dipole solitons can only propagate stably tens of micrometers. The number of the lines and the period of the grating can be controlled by adjusting the pulse area and the beam waist. Of particular interest is that a polarization grating emerges.

The outline of this paper is as follows: In Sec. II, the coupled set of Maxwell-Bloch equations is presented to model the interaction of the TE-polarized dipole soliton with the two-level medium. The numerical results are given and discussed in Sec. III. The final section is a summary of the main results.

II. MODEL

We consider the two-level medium driven by a two-dimensional high-order TE-polarized ultrashort pulse, in which the electric field is polarized along the x axis and is also assumed to be uniform along that axis, $\mathbf{E}(\mathbf{r},\mathbf{t})=\mathbf{E_x}(\mathbf{y},\mathbf{z},\mathbf{t})$. The Maxwell equations for the electric and magnetic field E_x, B_y, and B_z take the forms

$$\frac{\partial B_y}{\partial t}=-\frac{\partial E_x}{\partial z}, \tag{1a}$$

$$\frac{\partial B_z}{\partial t}=\frac{\partial E_x}{\partial y}, \tag{1b}$$

$$\frac{\partial D_x}{\partial t}=\frac{1}{\mu_0}\frac{\partial B_z}{\partial y}-\frac{1}{\mu_0}\frac{\partial B_y}{\partial z}, \tag{1c}$$

where the z axis is the propagation direction and the y axis is the transverse direction. The nonlinear response of the two-level medium is included through the relation $D_x=\epsilon_0 E_x+P_x$, where the macroscopic polarization $P_x=-Ndu$ is determined by the Bloch equations beyond the standard approximations SVEA and RWA,

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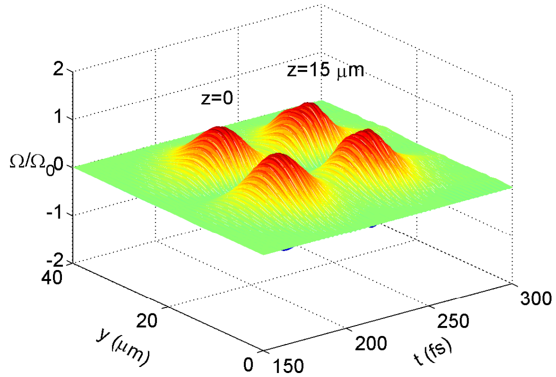


FIG. 1. (Color online) The out-of-phase dipole solitons of $\theta_p = 6\pi$ and $w_s = 6.0$ μm at the respective distances $z=0$ and 15 μm.

$$\frac{\partial u}{\partial t} = -\omega_{21}v - \gamma_1 u, \tag{2a}$$

$$\frac{\partial v}{\partial t} = \omega_{21}u - 2\frac{d}{\hbar}E_x w - \gamma_1 v, \tag{2b}$$

$$\frac{\partial w}{\partial t} = 2\frac{d}{\hbar}E_x v - \gamma_2(w - w_0). \tag{2c}$$

Here N is the density of the polarizable atoms, $\rho_{12}=(u+iv)/2$ is the off-diagonal density matrix element, $w=\rho_{22}-\rho_{11}$ is the population difference between the excited state $|2\rangle$ and the ground state $|1\rangle$, ω_{21} is the transition frequency, d is the dipole moment, w_0 represents the initial value of the population difference, and γ_1 and γ_2 are the polarization and population damping constants, respectively. $\Omega = dE_x/\hbar$ is the Rabi frequency. For simplicity, assume that the dipole moment d is aligned with the direction of polarization of the applied electric field $\mathbf{E}$, i.e., the x axis.

We employ a standard finite-difference time-domain (FDTD) [28] approach for solving the full-wave Maxwell equations, and a fourth-order Runge-Kutta method to solve the Bloch equations [21,27]. Asymmetric boundary conditions [29] are imposed at the transverse boundaries, which are placed far enough away so that the optical intensity is negligible. The region of medium is also assumed to be long enough such that the numerical reflection from the truncated end surface can be neglected. On the driving face ($z=0$) of the computational domain, the incident pulse, which is an out-of-phase dipole soliton, is defined as

$$E_x(y,z=0,t) = \sqrt{e}E_0\frac{y}{w_s}\exp\left(-\frac{y^2}{2w_s^2}\right)\mathrm{sech}[1.76(t-t_0)/\tau_p] \times\cos[\omega_p(t-t_0)]$$

[30], where E_0 is the peak input electric field and t_0 is the group delay. The choice of $t_0=200$ fs ensures that the field at $t=0$ is negligible. ω_p is the carrier frequency of the pulse and τ_p is the duration, i.e., the full width at half maximum (FWHM) of the intensity. Because the dipole soliton has a transverse structure of a Hermite-Gaussian mode, we define the beam waist w_s as the half of peak-to-peak separation of the intensity [30,31]. The field profile has zero intensity at $y=0$ and its intensity peaks off center at $y=\pm w_s$. The pulse area at its peak intensity is $\theta_p = \frac{dE_0}{\hbar}\tau_p\pi/1.76$. In the following analysis, we adopt the field and all material parameters based on Ref. [23]: $\tau_p=20$ fs, $\omega_p=\omega_0=2.3$ fs^{-1} ($\lambda=830$ nm), $d=2\times10^{-29}$ cm, $\gamma_1^{-1}=0.5$ ps, $\gamma_2^{-1}=1$ ps, and the density $N=1\times10^{24}$ m^{-3}. The system is initialized with $u=0$, $v=0$, and $w_0=-1$ at $t=0$. The considered intensities are less than the value of 4×10^{12} W/cm^2. For such laser intensity, recent experiments on semiconductors have demonstrated that a description in terms of two-level systems has been able to reproduce the experimental results amazingly well [32–34]. Moreover, a few significant works that have studied this model can also be found in Refs. [19,21,23]. So, when the laser intensity is of the order of 10^{12} W/cm^2, the simple two-level system can provide an adequate description [32].

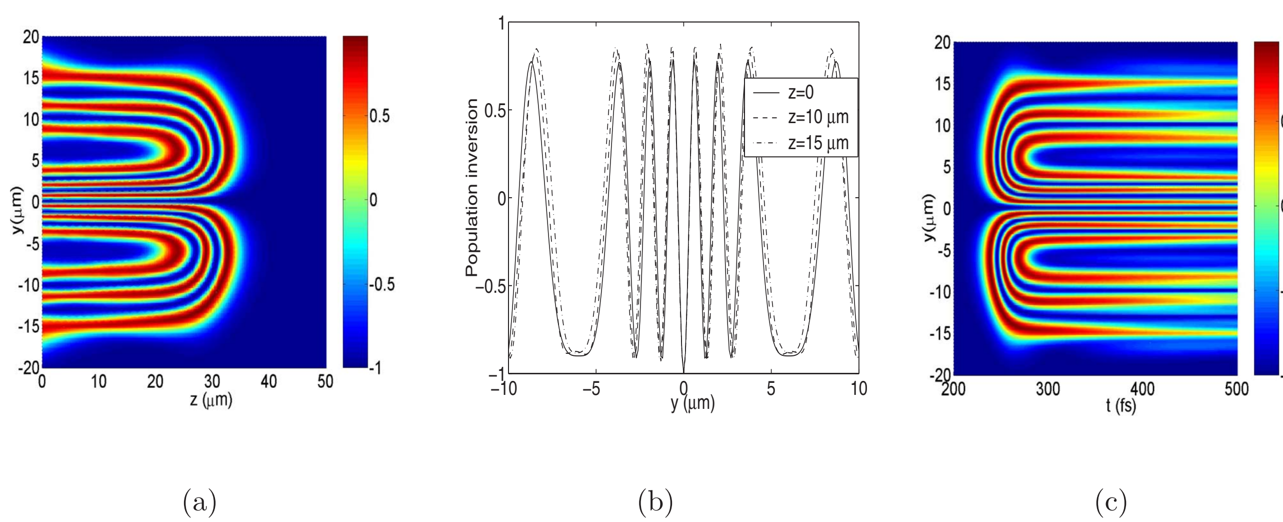
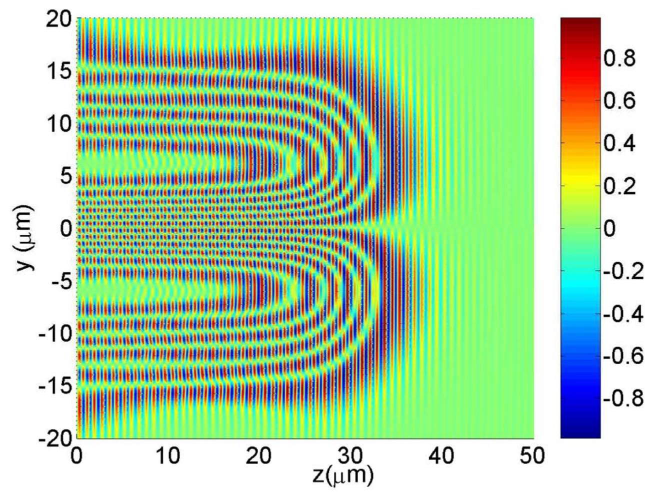


FIG. 2. (Color online) (a) Contour of the population inversion at $t=300$ fs. (b) The distribution of the population in the transverse direction at the positions of $z=0$, 10 μm, 15 μm. (c) Contour of the population inversion at $z=15$ μm.

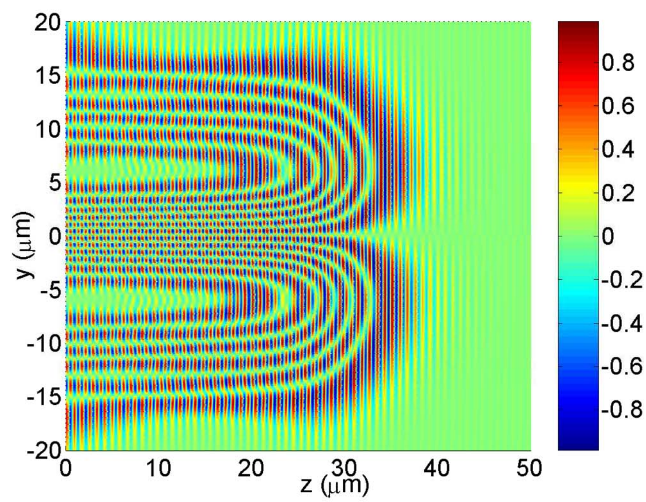
III. NUMERICAL RESULTS

In contrast to the case of one dimension, the transverse profile of the field crucially influences the propagation of the pulse and the response of the medium. We focus on the nonlinear response of the two-level atom driven by the $(1+1)$-dimensional dipole solitons of various peak intensities and beam waists but a fixed duration of 20 fs. When the dipole soliton is out of phase, its two components repel each other [13,31]. When the repulsive force of the soliton and the nonlinearity of the system compensate for or balance the diffraction, the soliton is stable. Moreover, for a propagating distance of tens of microns considered below, the influence of the diffraction is little. For an ultrashort dipole soliton of $\theta_p=6\pi$ and $w_s=6.0$ μm, it can be clearly seen from Fig. 1 that the diffraction is negligible.

According to the area theorem [11], for a pulse of area $k\pi$, the field completely inverses the medium for $w_0=-1$ when k is an odd integer, while when k is an even integer, the medium returns to its initial state after driven several complete Rabi flops. *So a population grating can be expected to emerge in the middle region after the pulse passes because the field varies continuously between the two peaks of the intensity in the transverse direction.* For the $2l\pi$ pulse, where l is an integer, the system undergoes $2l$ complete inversions in the transverse direction in the middle region. Consequently, the population grating has $2l$ lines. As an example, for the pulse shown in Fig. 1, a one-dimensional, six-line population grating is produced in the region of -5 $\mu\text{m}<y<5$ μm and $0<z<20$ μm [see Fig. 2(a)]. The population inversion w decreases slowly owing to the relaxation processes of the system. The interval of the central four lines is identical. It can be seen from Fig. 2(b) that the periods of the grating at the different positions $z=0$, 10 μm, 15 μm are all about 1.1 μm. Moreover, it can be clearly seen from Fig. 2(c) that the population grating can keep stable for about 200 femtoseconds. However, further investigations show that the grating degenerates after about half a picosecond. Importantly, a polarization grating that means a hybrid phase and amplitude grating is induced. The contours of the dispersive and absorptive components of the polarization at $t=300$ fs are illustrated in Fig. 3. Because the Maxwell-Bloch equations are solved without SVEA, the polarization oscillates in the propagation direction. The phase difference between the two components of the polarization is $\pi/2$. However, the more meaningful envelopes show a hybrid dispersive and absorptive grating. Moreover, the spatial structures of the dispersive and absorptive gratings are identical. The number of lines of the polarization grating is twice that of the population grating. The polarization grating can be qualitatively interpreted as follows. After the soliton passes, a long but very weak tail results from the fact that the medium still has energy residing in the dispersive component of the polarization which oscillates at ω_{21} (see Ref. [21]) if the incomplete RFs happen and the system is either completely inverted or in the ground state. In our cases, the polarization and population relaxations can be neglected because the duration of pulse is much less than the induced-dipole dephasing time and the population decay time. So, the relationship $u^2+v^2+w^2=1$ indicates a fact that the two components of the polarization oscillate at the frequency ω_{21} with the same magnitude at some positions where the population of the system is incompletely in either the ground state or the excited state. Furthermore, the magnitude of the polarization is finite when $|w|\neq 1$. While when $|w|=1$, the polarization is vanishing. The magnitude is maximum when $|w|=0$. Thus the period of the polarization grating is about half of that of population grating. Note that the population and polarization gratings both appear after the ultrashort dipole soliton almost passes through the medium. In our case, it is impossible to calculate the refractive index modulation depth and to clearly show the profiles of refractive index gratings because this medium does not exert phase and amplitude modulations on the ultrashort dipole soliton itself but on the following electric field. Furthermore, the perturbative expansion for the description of the material polarization response breaks down



(a)



(b)

FIG. 3. (Color online) Contours of the polarization at $t=300$ fs for the pulse of $\theta_p=6\pi$ and $w_s=6.0$ μm. (a) The dispersive component. (b) The absorptive component.

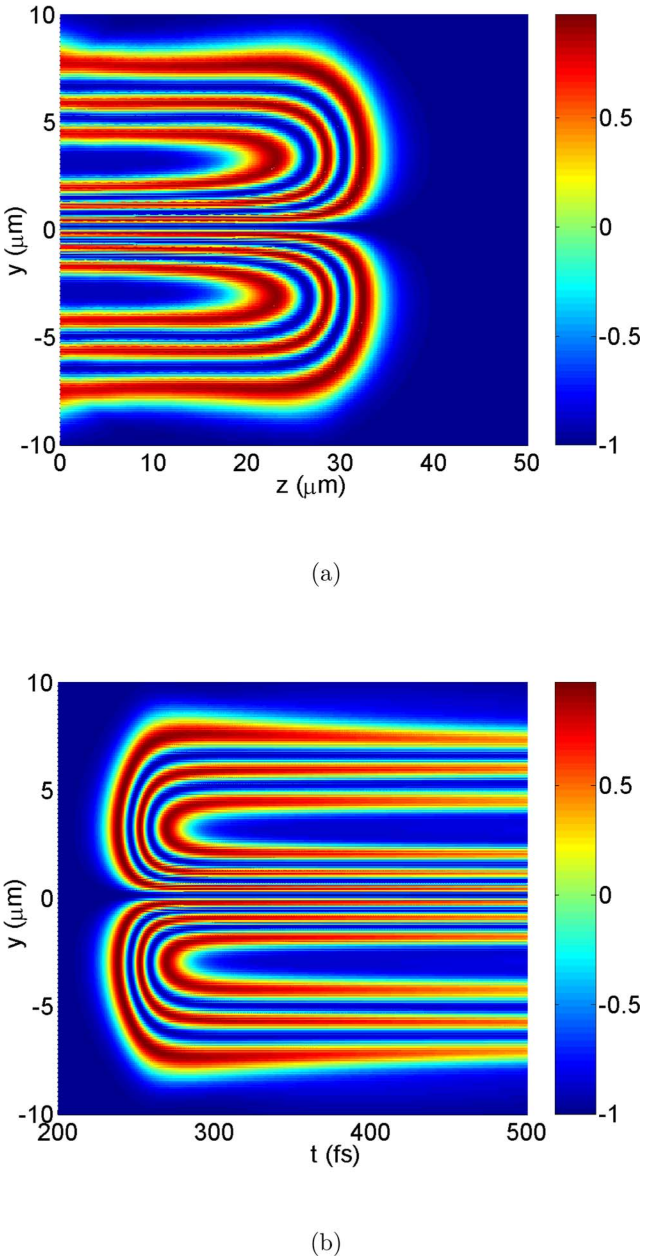


FIG. 4. (Color online) Contours of the population inversion for the pulse of $\theta_p=6\pi$ and $w_s=3$ μm. (a) Spatial distribution at $t=300$ fs. (b) Temporal dynamics at $z=15$ μm.

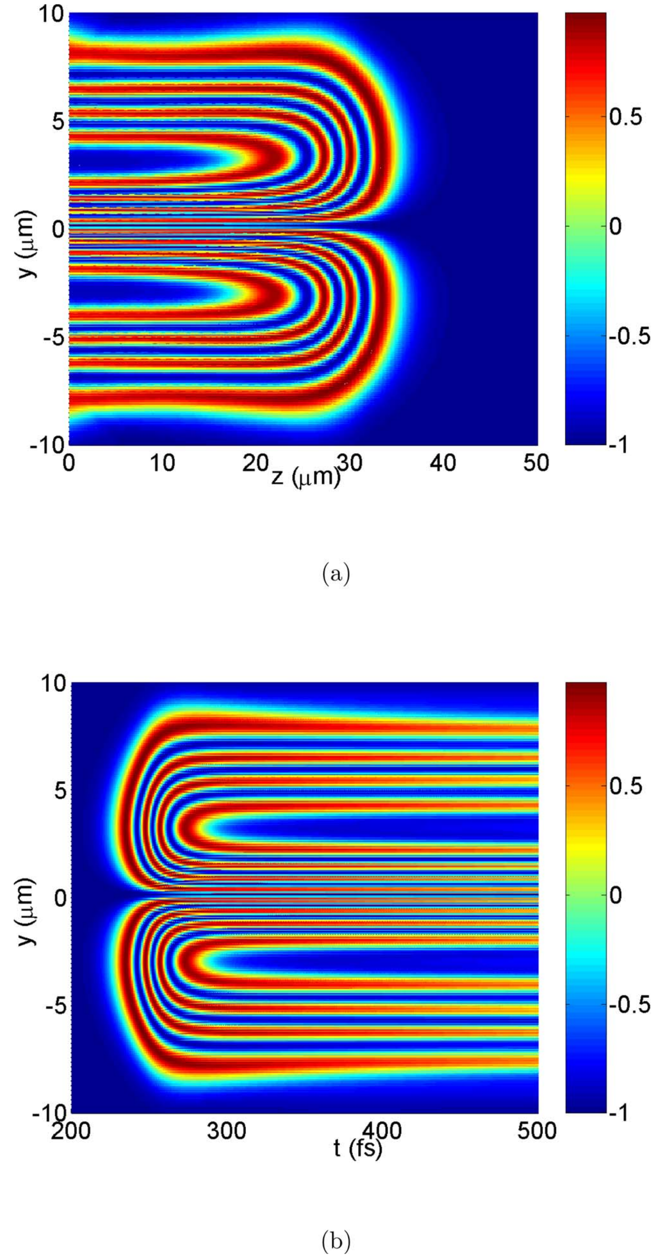


FIG. 5. (Color online) As in Fig. 4 but the pulse area is 8π. (a) Spatial distribution at $t=300$ fs. (b) Temporal dynamics at $z=15$ μm.

due to the very strong nonlinearities [35]. Nevertheless, maybe, it is feasible to investigate the refractive index grating with a weak probe beam which follows the dipole soliton. After developing an appropriate pump-probe technique, we will attempt to discuss the properties of the refractive index grating in the future work.

Further consideration is given to the control of the population and polarization gratings. For fixed pulse area, when the beam waist w_s of soliton decreases, the transverse interval between the two adjacent maximum population inversions become smaller. Thus, the grating period decreases as the beam waist decreases.

For the case of $\theta_p=6\pi$ and $w_s=3$ μm, the contours of the population inversion shown in Fig. 4 demonstrate a denser population grating compared with the case shown in Fig. 2. The grating period becomes 0.7 μm though the number of lines remains unchanged. In contrast, the population and polarization gratings have more lines and smaller space period as the pulse area increases. As illustrated by Fig. 5, for a pulse of $\theta_p=8\pi$ and $w_s=3$ μm, the population grating has eight lines parallelling z axis in the middle region. The central six lines of the grating are uniform and the interval between them becomes about 0.5 μm, which is less than one wavelength. Clearly, the population and polarization grating are significantly dependent on the beam waist and the area of pulse. Note that it is impossible to implement a subwavelength population or polarization grating by using a standing wave.

The response of the system to a dipole soliton of 100 fs or longer is also examined. In such a case, it takes a great amount of time for the pulse to pass the same medium. As a result, the population and polarization gratings are heavily distorted due to the relaxation of the system, the splitting, and the diffraction of the pulse. As for another extreme case, e.g., pulses with the durations less than a single optical cycle, the area theorem of nonlinear optics fails [36]. Consequently, the pulse cannot induce a uniform grating.

IV. CONCLUSION

In summary, by solving the $(1+1)$-dimensional full-wave Maxwell-Bloch equations, we have discussed the nonlinear response of the two-level medium to the dipole soliton. It is found that a new ultrafast transient population grating can be induced by an ultrashort dipole soliton. Particularly, a polarization grating appears. The numbers of lines of both gratings increases with an increase in the pulse area. The grating periods decrease as the pulse area increases or the beam waist decreases. Dipole solitons employed in this paper have been observed in experiments [31,37,38]. On the other hand, the technique to produce ultrashort pulses with beam waist about one wavelength has been demonstrated [39,40]. So, an ultrafast transient grating can be practically designed based on this coherent control scheme.

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