Ultrafast transient ring-shaped population grating

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We demonstrate an ultrafast transient, ring-shaped population grating induced by an ultrashort hollow Gaussian laser bullet by solving the three-dimensional full-wave Maxwell-Bloch equations. Through adjusting the beam waist and the area of the pulse, we can control the number of lines and the period of the grating. Based on this coherent control scheme, a door to produce gratings with complex transverse structure is opened.

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In recent years, there has been considerable interest in population gratings (PGs) due to their several possibilities of applications, such as laser diffraction on electromagnetically induced grating [1], transient Bragg diffraction [2], and high-order diffraction in transient four-wave mixing with semiconductors [3]. Brown *et al.* have studied the molecular dynamics with femtosecond transient grating techniques [4]. The population grating techniques have been used to characterize CdTe quantum dots grown on ZnSe [5]. More recently, Scholes, Kim, and Wong have measured the exciton spin relaxation in quantum dots by using ultrafast transient polarization grating spectroscopy [6]. The relation between the phase and amplitude components of an inverse population grating has been well discussed [7]. Stepanov and co-worker have theoretically and experimentally studied the intensity dependence of the transient two-wave mixing of two cw by population grating in the Er-doped fiber [8]. Lately, we have demonstrated an ultrafast transient population grating by using a two-dimensional (2D) ultrashort dipole soliton [9]. Although PGs have been well studied, they are plane gratings. It is hardly to realize a ring-shaped population grating as demonstrated below in 2D space or by a standing-wave field as works mentioned above.

Under the condition that the laser beam is assumed to be a plane wave front, it has been proved that one-dimension Maxwell-Bloch equations are a powerful tool for the investigation of the coherent laser-matter interactions [10,11]. In contrast, in the case of laser beams of finite transversal diameters, the transverse effects play a critical role in the response of the system [12]. Based on the coupled Maxwell-Bloch equations with the slowly varying envelope approximation (SVEA) and rotating-wave approximation (RWA), the influence of transverse effects on the self-focusing or induced focusing of coherent optical pulses have been investigated [13–16]. Manassah and Gross have reported induced focusing in a Λ system by analysis the 3D Maxwell-Bloch equations with SVEA [16]. While for ultrashort laser pulses, the limitations of SVEA are reported by a number of works [17–21]. Furthermore, few works have investigated the transverse effects of laser pulses under considering one transverse dimension beyond SVEA and RWA. Forysiak, Moloney, and Wright have reported the nonlinear focusing of femtosecond pulses due to the self-reflection on a saturable absorbing interface [22]. Eisenberg and Silberberg numerically observed edge-type phase defects during the self-focusing of ultrashort pulses [23]. Slavcheva *et al.* have investigated the self-induced transparent (SIT) of the two-dimension TM and TE guided modes in the parallel-plat mirror optical waveguide [24]. However, to the best of our knowledge, the transverse effects beyond SVEA and RWA in a three-dimensional space have not yet been discussed.

By using an ultrashort hollow Gaussian laser bullet (HGLB) [25,26], we demonstrate an ultrafast transient, ring-shaped population grating based on the three-dimensional (3D) full-wave Maxwell-Bloch equations beyond SVEA and RWA. We consider a two-level medium driven by a three-dimensional ultrashort HGLBs. For simplicity, assume that the applied linearly polarized electric field is polarized along the x axis and the medium is only polarizable in this direction. The Maxwell equations for the electric and magnetic fields E_x, E_y, E_z and B_x, B_y, B_z take the forms

$$\frac{\partial B_x}{\partial t} = -\frac{\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z}, \tag{1a}$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}, \tag{1b}$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y}, \tag{1c}$$

$$\frac{\partial D_x}{\partial t} = \frac{1}{\mu_0}\frac{\partial B_z}{\partial y} - \frac{1}{\mu_0}\frac{\partial B_y}{\partial z}, \tag{1d}$$

$$\frac{\partial D_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial B_z}{\partial x} - \frac{1}{\mu_0}\frac{\partial B_x}{\partial z}, \tag{1e}$$

$$\frac{\partial D_z}{\partial t} = \frac{1}{\mu_0}\frac{\partial B_y}{\partial x} - \frac{1}{\mu_0}\frac{\partial B_x}{\partial y}, \tag{1f}$$

where the z axis is the propagation direction, $D_y = \varepsilon_0 E_y, D_z = \varepsilon_0 E_z$. The nonlinear response of the two-level medium is included through the relation $D_x = \epsilon_0 E_x + P_x$, where the mac-

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roscopic polarization $P_x = -N\,du$ is determined by the Bloch equations without SVEA and RWA

$$\frac{\partial u}{\partial t} = -\omega_0 v - \gamma_1 u, \tag{2a}$$

$$\frac{\partial v}{\partial t} = \omega_0 u - 2\frac{d}{\hbar}E_x w - \gamma_1 v, \tag{2b}$$

$$\frac{\partial w}{\partial t} = 2\frac{d}{\hbar}E_x v - \gamma_2 (w - w_0). \tag{2c}$$

Here $\rho_{12}=(u+iv)/2$ is the off-diagonal density matrix element, $w=\rho_{22}-\rho_{11}$ is the population difference between the excited state $|2\rangle$ and the ground state $|1\rangle$, N is the density of the polarizable atoms, ω_0 is the transition frequency, d is the dipole moment, w_0 represents the initial value of the population difference, and γ_1 and γ_2 are the polarization and population damping constants, respectively. $\Omega = dE_x/\hbar$ is the Rabi frequency. The method of the simulation is introduced in Refs. [9,19,22]. On the driving face ($z=0$) of the computational domain, the incident pulse, which is hollow Gaussian distribution in the transverse directions, is defined as $E_x(x,y,z=0,t)=(e/n)^n E_0 (r^2/w_s^2)^n \exp(-r^2/w_s^2) \times \mathrm{sech}[1.76(t-t_0)/\tau_p]\cos[\omega_p(t-t_0)]$ [25,26], where $r = \sqrt{x^2+y^2}$, E_0 is the peak input electric field, n is the order of HGLB, and t_0 is the group delay. τ_p is the duration, i.e., the full width at half maximum (FWHM) of the intensity. The choice of $t_0 = 50$ fs ensures that the field at $t=0$ is negligible. ω_p is the carrier frequency of the pulse. The beam waist w_s is the waist size of the Gaussian beam [25,26]. The pulse is a laser bullet with zero central intensity along the beam axis. The intensity profile is maximal at the location of $r=\sqrt{n}w_s$. The pulse area at its peak intensity is $\theta_p = (dE_0/\hbar)\tau_p \pi/1.76$. In the following analysis, we adopt the field and all material parameters based on Ref. [20]: $\tau_p = 20$ fs, $\omega_p = \omega_0 = 2.3\ \mathrm{fs}^{-1}(\lambda = 830\ \mathrm{nm})$, $d = 2\times 10^{-29}$ C m, $\gamma_1^{-1} = 0.5$ ps, $\gamma_2^{-1} = 1$ ps, and the density $N = 1\times 10^{24}\ \mathrm{m}^{-3}$. The system is initialized with $u=0$, $v=0$, and $w_0=-1$ at $t=0$.

Different from the case of one dimension, the transverse effects of laser-matter interactions have crucially influences on the response of the medium. Here, we demonstrate a ring-shaped population grating by exploiting a 3D hollow Gaussian laser bullet with fixed duration of 20 fs. For the pulse parameters used in the calculation, the Rayleigh range Z_R is larger than 240 μm. The hollow Gaussian beam has good propagation stability when the propagating distance is less than the Rayleigh range Z_R [25]. Thus, within a propagating distance of ten microns considered below, the diffraction and the divergence of the pulse are slightly.

According to the area theorem [10] and our analysis in Ref. [9], an ultrafast transient, ring-shaped population grating can be expected to emerge after the HGLB passes. The order n only influences substantially on the central zone where the system is in the ground state because only the dark zone of the beam dependent on it [25]. So, we employ a HGLB with a constant order of 3. Figure 1 shows a ring-shaped population grating induced by a HGLB of $\theta_p = 8\pi$ and $w_s = 10$ μm. Clearly, the population grating has eight homocentric lines in the region of 8 μm $< r <$ 28 μm. It means that the medium inverses eight times in the radial direction. It can be seen from Fig. 2, the population grating is identical in the propagating direction. All of the periods of population grating at the different positions $z=0$, 5, and 10 μm are about 2.8 μm. The medium in the middle zone is nearly in the ground state because the beam is dark in this zone. Further investigation shows that the grating can keep stable for about 150 fs although it degenerates after about half a picosecond owing to the relaxation processes.

Next, we focus on the control of the grating. For constant pulse area, the field varies faster as the beam waist w_s decreases and the interval between the two adjacent maximal population inversions become smaller. As a result, the period of grating decreases with decreasing the beam waist. For a laser bullet of $\theta_p = 8\pi$ and $w_s = 8$ μm, the contour of the population inversion shown in Fig. 3 illustrates a denser grating compared with the case in Fig. 1. The number of lines remains unchanged, but the period of population grating becomes 2.2 μm. In contrast, the population grating has more lines and smaller spacial period as the pulse area increases. For a HGLB of $\theta_p = 10\pi$ and $w_s = 10$ μm, the population grating has ten homocentric lines, as shown in Fig. 4. The inter-

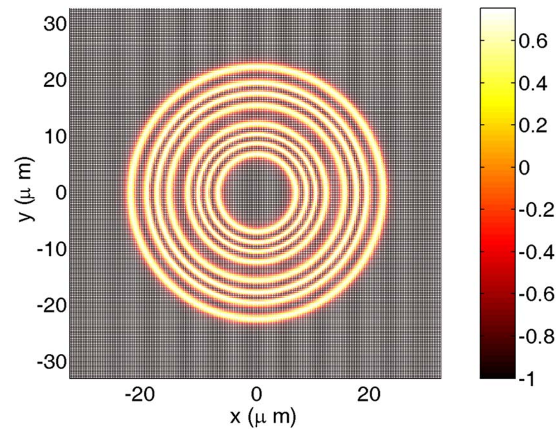

FIG. 1. (Color online) Contour of the population grating at the location $z=10$ μm and the instant of $t=200$ fs for $\theta_p = 8\pi$ and $w_s = 10$ μm.

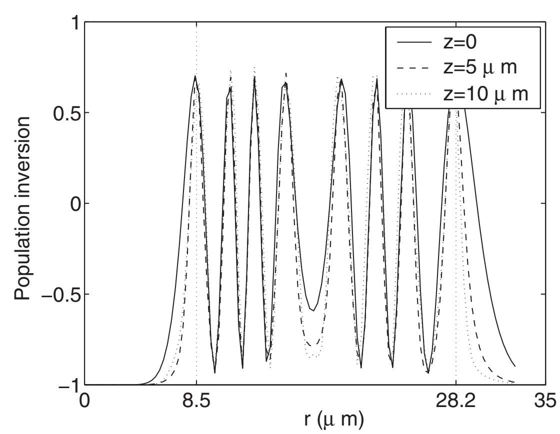

FIG. 2. Distribution of the population in the radial direction at the positions of $z=0$, 5 μm, and 10 μm at $t=200$ fs.

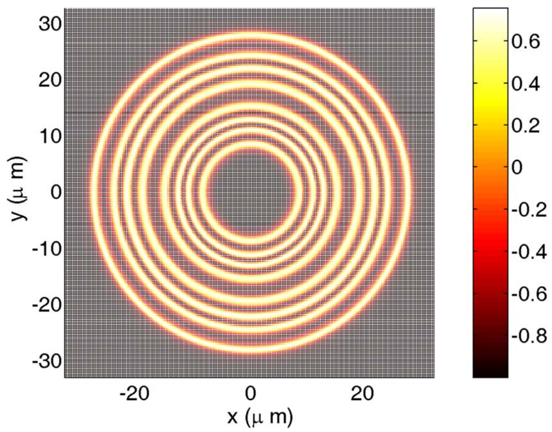

FIG. 3. (Color online) As in Fig. 1, but $w_s = 8\ \mu$m.

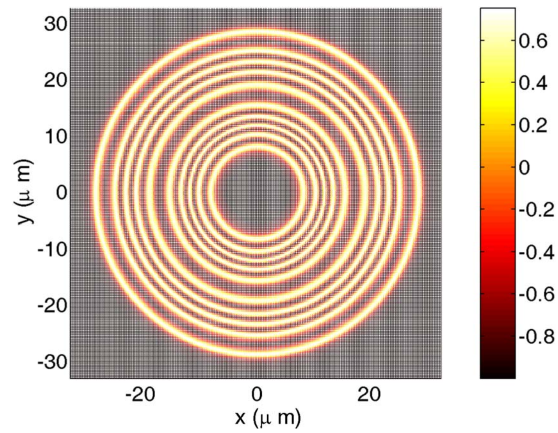

FIG. 4. (Color online) As in Fig. 1, but $\theta_p = 10\pi$.

val between these lines are uniform and becomes about 2.3 μm. Obviously, the beam waist and the area have crucially influences on the population grating.

In summary, we have investigated the nonlinear response of the two-level medium to the HGLBs numerically. We found that a ring-shaped ultrafast transient population grating can be induced by an ultrashort HGLB. The number of lines of population grating increases with increasing the pulse area. The grating periods decrease as the pulse area increases or the beam waist decreases. Hollow Gaussian beams have been well investigated theoretically [25,27] and observed in experiments [26]. On the other hand, the technique to produce ultrashort pulses with beam waist about one wavelength has been demonstrated [28,29]. Hence our scheme will be an efficient way to produce complex ultrafast transient grating by designing the transverse profile of the laser beam.

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