Ground-state cooling of a nanomechanical resonator coupled to two interacting flux qubits

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We discuss cooling of a nanomechanical resonator to its mechanical ground state by coupling it to a collective system of two interacting flux qubits. We find that the collectivity crucially improves cooling by two mechanisms. First, cooling transitions proceed via subradiant Dicke states, and the reduced linewidth of these subradiant states suppresses both the scattering and the environmental contribution to the final phonon number. Second, detrimental carrier excitations without change in the motion of the resonator are suppressed by collective energy shifts.

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I. INTRODUCTION

Micromechanical and nanomechanical resonators (NAMRs) are currently in the focus of interest because of the fascinating applications they promise.[1] Access to these applications requires cooling of the mechanical motion to the quantum-mechanical ground state. Different routes toward the ground state have been suggested. For example, backaction cooling or sideband cooling in optomechanical systems has been proposed and demonstrated via cavity-assisted radiation pressure.[2–6] In the electromechanical domain, cooling methods have been suggested which are based on auxiliary devices such as superconducting resonators,[7–9] superconducting qubits,[10,11] superconducting single-electron transistors,[12] or quantum dots.[13,14] Backaction cooling or sideband cooling can only reach the ground state if the mechanical resonance frequency exceeds the bandwidth of the cavity or auxiliary quantum system. However, a recent experiment in this strong confinement regime (SCR) (Ref. 8) found an unclear source of excess fluctuations, which prohibited access to the ground state by lowering the environmental temperature, in contrast to theoretical predictions.[6] Furthermore, the SCR restricts the accessible parameter range, and is often difficult to implement. Active feedback cooling[4,15,16] is a candidate for overcoming this limit. However, it typically requires difficult control and precise measurement of displacement of the resonator. Another route is to make use of quantum interference. In Ref. 17, a system was proposed in which the mechanical displacement of the oscillator changes the damping, rather than the frequency of a coupled cavity. This leads to destructive interference in the noise contributions, and thus to the possibility of ground-state cooling already outside the SCR. Recently, we analyzed a cooling scheme for NAMR based on electromagnetically induced transparency (EIT).[18] EIT cooling has originally been proposed[19] and demonstrated[20] for trapped ions, and works by eliminating unwanted heating transitions via destructive quantum interference.

Here, we propose an efficient ground-state cooling scheme for a NAMR without counterpart in the cooling of atoms or ions. The NAMR is coupled to two interacting flux qubits as shown in Fig. 1. The mutual interaction of the qubits gives rise to frequency-shifted collective qubit states with modified decay rates. We find that in a suitable cooling-field configuration, the collective frequency shift effectively eliminates detrimental qubit carrier excitations without change in the NAMR motion. At the same time, cooling via the subradiant collective qubit state suppresses both contributions from the thermal environment and the scattering of the cooling field to the cooling limit. Due to these collective effects, our scheme offers efficient ground-state cooling already for rather small driving fields, and for NAMR operating outside the strong-confinement regime. An implementation is further assisted by the simple level structure of two two-level qubits operating close to the optimum point. Unlike in backaction cooling and feedback cooling, the final NAMR state has no coherent shift in its phonon number.

II. ANALYTICAL CONSIDERATIONS

A. Physical picture

We start by explaining the physical mechanisms behind collectivity-assisted cooling. First, we consider the two interacting qubits without coupling to the motional state $|n\rangle$ of the oscillator.[21] Each qubit $j(j \in \{L,R\})$ is a two-level quantum system with ground state $|g_j\rangle$, excited state $|e_j\rangle$, and decay rate γ. The two qubits L and R are coupled via their mutual inductances, such that the system dynamics can conveniently be described in the collective-state basis

$$|e\rangle = |e_L e_R\rangle, \tag{1a}$$

$$|s\rangle = (|e_L g_R\rangle + |g_L e_R\rangle)/\sqrt{2}, \tag{1b}$$

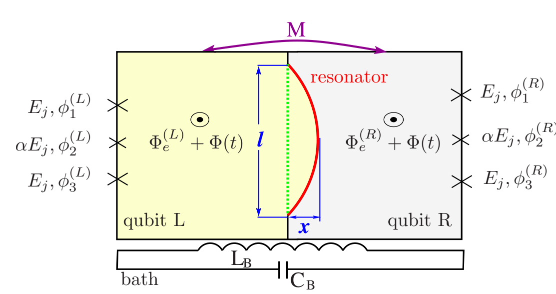

FIG. 1. (Color online) A nanomechanical resonator coupled to two flux qubits. The flux qubits interact with each other through their mutual inductance M and are damped via a common bath modeled as a LC circuit.

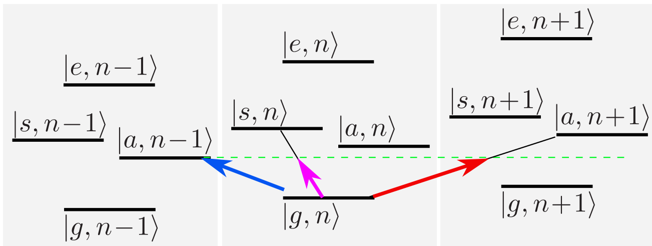

FIG. 2. (Color online) Energy level diagram for the NAMR coupled to two interacting flux qubits. Cooling (heating) transitions $|n\rangle \to |n-1\rangle$ ($|n\rangle \to |n+1\rangle$) proceed via the narrow collective subradiant state $|a\rangle$ and are denoted by the blue/left (red/right) arrow. Carrier excitation $|n\rangle \to |n\rangle$ (purple arrow) is possible only via the strongly detuned symmetric collective state.

$$|a\rangle = (|e_L g_R\rangle - |g_L e_R\rangle)/\sqrt{2}, \tag{1c}$$

$$|g\rangle = |g_L g_R\rangle. \tag{1d}$$

In a strongly interacting system,[22,23] the antisymmetric state $|a\rangle$ is characterized by a significantly reduced decay rate $\Gamma_a \ll \gamma$ while the symmetric state $|s\rangle$ has an enhanced rate $\Gamma_s \approx 2\gamma$. Moreover, the two states are shifted in energy due to the qubit-qubit interactions and inductance coupling. Analogously, a driving field which is applied symmetrically to the two qubits will couple to transitions $|g,n\rangle \leftrightarrow |s,n\rangle$ and $|s,n\rangle \leftrightarrow |e,n\rangle$ but not to those via $|a,n\rangle$.

Next, we consider in addition the coupling to the motional eigenstates $|n\rangle$ of the NAMR. A relevant part of the total-energy level spectrum is shown in Fig. 2. It turns out that the symmetrically applied driving field still couples the ground state $|g,n\rangle$ to the symmetric state $|s,n\rangle$ without change in the motion but it couples $|g,n\rangle$ to the antisymmetric states $|a,n\pm 1\rangle$ if the motional state increases or decreases. This is because a movement of the oscillator always increases the loop area of one of the qubits while decreasing the area of the other. The three leading excitation channels are depicted in Fig. 2 for a field in resonance with the red sideband to state $|a,n-1\rangle$. Then, as in sideband cooling,[13] this leads to a cooling of the NAMR but with substantially improved performance. The first reason for this is that ground-state cooling is only possible in the SCR in which the NAMR eigenfrequency ν exceeds the scattering state width. Thus, in regular sideband cooling, $\gamma/\nu < 1$ is required. In contrast, collectivity-assisted cooling relies on scattering via subradiant states, leading to the condition $\Gamma_a/\nu < 1$. In particular, in the common case $\Gamma_a \ll \nu < \gamma$, a single qubit would be in the nonresolved regime whereas the collective system effectively becomes resolved. The second advantage is a suppression of carrier excitations $|n\rangle \to |n\rangle$ due to the always-on coupling from the mutual inductance M. Carrier excitations on average lead to a heating of the resonator and provide a lower limit to the final phonon number.[24] This suppression is achieved mainly by the large always-on coupling induced energy shift of the collective states $|s\rangle$ and $|a\rangle$, which moves the symmetric carrier scattering channel out of resonance with the cooling field. Third, the narrow width Γ_a also suppresses nonresonant heating processes $|n\rangle \to |n+1\rangle$ as compared to the single-qubit case. These advantages together lead to a greatly improved collectivity-assisted cooling performance.

B. Model

We now proceed with a quantitative analysis. The NAMR has effective mass $M_{\rm eff}$, length l, frequency ν, and quality factor Q. It can be treated as a harmonic quantum resonator with Hamiltonian

$$H_r = \hbar \nu b^\dagger b.$$

The quantized displacement operator is $x = X_0(b+b^\dagger)$, where b is the annihilation operator. $X_0 = \sqrt{\hbar/2M_{\rm eff}\nu}$ denotes the zero-point fluctuation of the NAMR. Each flux qubit consists of superconducting loops with three Josephson junctions, two identical and one junction smaller by a factor α. The qubits are exposed both to a constant magnetic field $\mathbf{B}$ perpendicular to the plane and to a common driving microwave field $\Phi(t)$, which induces a time-dependent magnetic flux (TDMF). Following a standard treatment of the coupled flux qubits,[25] both qubits are modeled as two-level systems. In the collective-state basis, the free system Hamiltonian of flux qubits and NAMR is given by

$$H_0 = \hbar \nu b^\dagger b + \hbar \Delta (R_{ee} - R_{gg}) + \hbar \Lambda (R_{ss} - R_{aa}), \tag{2}$$

where the qubit operators are defined as $R_{jk} = |j\rangle\langle k|$ $(j,k \in \{e,s,a,g\})$. The detuning $\Delta = \omega_0 - \omega_L$ is the difference of the qubit transition frequency ω_0 and the TDMF frequency ω_L. Λ includes the always-on inductance coupling and the collective shifts induced by the dipole-dipole coupling of the two qubits.[25,26] The qubits operate near their optimal point $f=0.5$, and are assumed degenerate, which is reasonable if their transition energy difference is much smaller than Λ. We assume that the qubit transition frequencies are large enough to neglect thermal excitations.

The qubit-NAMR interaction is modeled by its interaction Hamiltonian reading as

$$H_I = \sqrt{2}\hbar\Omega (R_{es} + R_{sg} + R_{se} + R_{gs}) + \sqrt{2}\hbar\Omega\eta (b+b^\dagger)(R_{ag} + R_{ga} - R_{ea} - R_{ae}), \tag{3}$$

where the Rabi frequencies are given by

$$\hbar\Omega_j = \frac{\alpha E_J}{2}\frac{2\pi}{\Phi_0} A \mathcal{S}_j \tag{4}$$

with coupling energy αE_J, and we define $\hbar\Omega_L = \hbar\Omega_R = \hbar\Omega$. Further,

$$\mathcal{S}_j = \langle e_j | \sin(2\varphi_p^{(j)} + 2\pi f) | g_j \rangle, \tag{5a}$$

$$\mathcal{C}_j = \langle e_j | \cos(2\varphi_p^{(j)} + 2\pi f) | g_j \rangle \tag{5b}$$

with phase $\varphi_p^{(j)} = (\varphi_1^{(j)} - \varphi_2^{(j)})/2$. A contribution from the momentum can be neglected. The Lamb-Dicke parameters evaluate to

$$\eta_j = \varsigma_j B l X_0 2\pi/\Phi_0,$$

where $\varsigma_j=\mathcal{C}_j/\mathcal{S}_j$. We assume that $\eta=\eta_L=\eta_R$. Our system couples to an environment of temperature T, such that the thermal NAMR occupation is evaluated by

$$N_i = [\exp(\hbar\nu/k_B T) - 1]^{-1}. \tag{6}$$

To model our system with local fluctuations such as $1/f$ noise, we introduce phenomenologically a pure dephasing to describe the nonMarkovian dynamics but treat the relaxations in Born-Markovian approximation. Again, using the rotating wave, and Lamb-Dicke approximations, the master equation is

$$\dot{\rho} = -\frac{i}{\hbar}[H_0 + H_I,\rho] + \mathcal{L}_{SE}\rho + \mathcal{L}_{\phi}\rho + [N_i+1]\mathcal{L}(\nu/Q,b)\rho + N_i\mathcal{L}(\nu/Q,b^{\dagger})\rho, \tag{7a}$$

$$\mathcal{L}_{\phi} = \sum_j \mathcal{L}(\Gamma_{\phi}/2,|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|), \tag{7b}$$

$$\begin{aligned}\mathcal{L}_{SE} = {} & \mathcal{L}(\Gamma_s,R_{se}+R_{gs}) + \mathcal{L}(\Gamma_a,R_{ae}-R_{ga}) + \mathcal{L}[\Gamma_s\eta^2,(R_{ae} - R_{ga})(b+b^{\dagger})] + \mathcal{L}[\Gamma_a\eta^2,(R_{se}+R_{gs})(b+b^{\dagger})] \\ & + \mathcal{L}[\Gamma_s\eta,(R_{ga}-R_{ae})(b+b^{\dagger}),R_{se}+R_{gs}] \\ & + \mathcal{L}[\Gamma_a\eta,(R_{se}+R_{gs})(b+b^{\dagger}),R_{ga}-R_{ae}],\end{aligned} \tag{7c}$$

$$\mathcal{L}(\kappa,A)\rho = \frac{\kappa}{2}\{2A\rho A^{\dagger} - [A^{\dagger}A,\rho]_+\}, \tag{7d}$$

$$\mathcal{L}(\kappa,A_1,A_2)\rho = \frac{\kappa}{2}\sum_{i\neq j}\{2A_i\rho A_j^{\dagger} - [A_j^{\dagger}A_i,\rho]_+\}. \tag{7e}$$

The modified damping coefficients $\Gamma_s=\gamma+\gamma_{12}$ and $\Gamma_a=\gamma-\gamma_{12}$ indicate the creation of subradiant and superradiant states due to the coupling with the bath. Such subradiant states have been observed experimentally.[27,28] We derive the relaxation rates of superconducting qubits coupled to a transmission line with impedance Z by a mutual inductance M_D modeled as an one-dimensional open space. The relaxation rate of an isolated qubit takes the form

$$\gamma = \sin^2(\theta)(M_D I_p)^2\omega_0/(\hbar Z), \tag{8}$$

where $\sin(\theta)=t/\omega_0$ with tunneling t,[29] and $\kappa=\nu/Q$. This formula is in agreement with previous results.[29,30] The dipole-dipole cross damping rate γ_{12} evaluates to

$$\gamma_{12} = \gamma\cos(\kappa r_{12}), \tag{9}$$

where κ is the wave number corresponding to the transition frequency and r_{12} is the effective separation between the two qubits. For example, an effective distance of about $\lambda/20$ (Ref. 23) leads to $\gamma_{12}=0.95\gamma$. The individual pure dephasing of the qubits with rate Γ_{ϕ} is phenomenologically described by $\mathcal{L}_{\phi}\rho$. The combination of a Markovian master equation with a phenomenological pure dephasing is widely used to model the quantum dynamics of superconducting qubits in good agreement with experiments.[27,30,31] The last two terms in Eq. (7a) result from the coupling of NAMR and thermal environment.

As we start cooling from thermal equilibrium with initial phonon number $N_i=N(\nu)$, the final phonon number n_{ss} is determined contributions from the thermal environment and from the cooling-field scattering,

$$n_{ss} = C(\eta,\Omega,\Gamma_a)\Gamma_a\nu\frac{N_i}{(\eta\Omega)^2 Q} + G(\eta,\Omega,\Gamma_a)\left(\frac{\Gamma_a}{4\nu}\right)^2. \tag{10}$$

Here we introduce coefficients $C(\eta,\Omega,\Gamma_a)$ and $G(\eta,\Omega,\Gamma_a)$ to accommodate the deviation of our approximate analytical expression from the numerical results. The analytical parts of Eq. (10) allow for a comparison with other cooling methods. We find that the environmental contribution to the cooling limit is suppressed by a factor of $C\Gamma_a\nu$ in comparison with sideband cooling[13] and backaction cooling.[6] For this comparison, we assume effective Lamb-Dicke parameters defined as $\eta_{\text{eff}}=\eta\Omega/\gamma$ in our case and $\eta_{\text{eff}}=\eta\sqrt{n_{\max}}$ in the backaction cooling scheme, where $\eta=(d\omega_c/dx)(X_0/\nu)$ with the cavity resonance frequency ω_c and $n_{\max}$ as the photon number on resonance.[6] Interestingly, as the environmental contribution is proportional to ν, the nonresolved regime with small ν is favorable. We also find that in our cooling scheme there is no coherent shift in the final phonon number, other than in backaction cooling[6] and feedback cooling.[16] This is crucial for many applications such as in quantum information science.[32]

III. NUMERICAL ANALYSIS

A. Parameters

We now turn to a full numerical study of Eq. (7). We envisage typical flux qubit parameters α=0.58, E_J/E_C =200, $E_J/2\pi\sim190$ GHz ($I_C\sim380$ nA), where E_J and E_C are the Josephson and charging energies of junction, respectively. We choose a bias flux f_b=0.505, and calculate a transition frequency $\omega_0/2\pi\sim6$ GHz, and a transition matrix element $\mathcal{S}_j\sim-0.49$ yielding $\varsigma\sim0.25$. The decay rate γ evaluates to about $2\pi\times2$ MHz, and experimental results show that this rate can remain similar over a wide range of bias[33] and could be engineered.[30] For a typical noise spectral density with magnitude $A=(1\,\mu\Phi_0)^2$,[33] the pure dephasing Γ_{ϕ} has a maximum value of $2\sqrt{A\ln 2}I_p/\hbar\sim0.42\gamma$,[29] which occurs at large bias. For our bias choice f_b=0.505, smaller dephasing can be expected. Nevertheless, to accommodate for other sources of noise, we assume $\Gamma_{\phi}=0.5\gamma$. Regarding the resonator, a double clamped NAMR with length of 300 μm and transversal area 50 nm$\times$40 nm made from silicon nitride can be manufactured with frequency $\nu\approx2\pi\times1$ MHz and quality $Q\approx10^6$.[34] This NAMR leads to $\eta\sim0.003$ for $B\sim150$ mT, compatible with Nb-based superconducting qubits. Note that critical magnetic field B_c of superconducting nanowires with lateral dimensions smaller than the coherence length is much higher than that of bulk materials.[35,36] Thus, Al based superconducting qubits could also be used to implement our scheme.

The sample chip could be placed in a dilution refrigerator of 20 mK giving rise to an initial phonon number $N_i\sim400$.

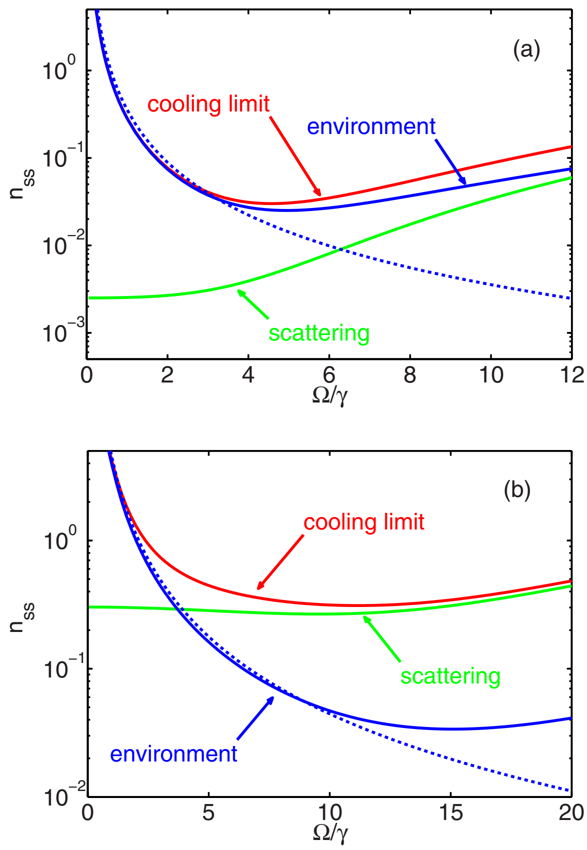

FIG. 3. (Color online) Final average phonon number n_{ss} and its contributions as a function of the cooling-field Rabi frequency Ω. Parameters are $N_i=400$, $Q=10^6$, $\Gamma_a=0.1\gamma$, and $\eta=3\times10^{-3}$. Dephasing is (a) $\Gamma_\phi=0$ and (b) $\Gamma_\phi=0.5\gamma$. A fit to the environmental contribution by Eq. (10) is shown as dashed line, with (a) $C=0.16$ and (b) $C=2$.

Assuming that the qubits are driven by a typical 50 Ω microwave line coupled by mutual inductance $\sim$8.3 pH, a Rabi frequency of 10γ only requires $\sim$0.2 pW. This small input power helps to reduce nonlinear and phase noise.[7] Throughout our investigation, we fix $\Lambda=500\gamma$ and apply the TDMF with detuning $\Delta=\nu+\Lambda$ corresponding to a red sideband excitation $|g,n\rangle\rightarrow|a,n-1\rangle$. Couplings Λ of this order have been observed experimentally based on large mutual inductance.[21] The large value of the always-on coupling at the same time reduces the influence of inevitable differences between the two qubits in an actual implementation on the cooling limit.

B. Results

An example for the dependence of the final phonon number n_{ss} on the TDMF strength Ω is shown in Fig. 3. Without pure dephasing, one can ideally cool a NAMR from $N_i=400$ to a final phonon number n_{ss} smaller than 0.05 in the range of $3\le\Omega/\gamma\le8$, see Fig. 3(a). This corresponds to a ratio N_i/N_f of initial to final phonon number exceeding 10^3. By fitting the environmental contribution, we find that $C=0.16$. Our analytic formula (10) becomes invalid for Ω exceeding 4γ. Taking into account the pure dephasing $\Gamma_\phi=0.5\gamma$, we find that n_{ss} is still lower than 0.5 in the range of $4\le\Omega/\gamma\le20$, see Fig. 3(b). At $\Omega=10\gamma$, the ground-state occupation is 76%. Further investigation show that ground-state cooling is possible as long as $\Gamma_\phi<\gamma$.

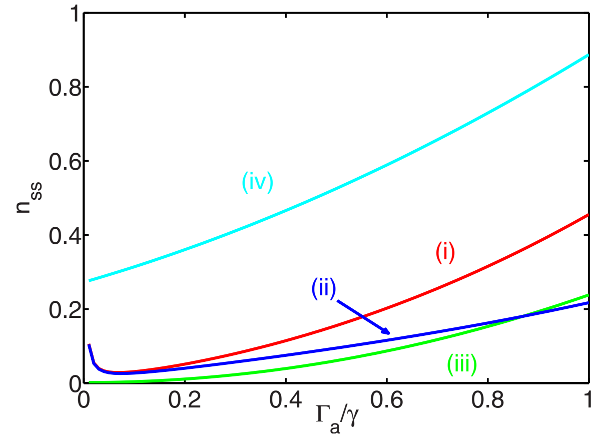

FIG. 4. (Color online) The final resonator phonon number n_{ss} versus the subradiant decay width Γ_a. Curves (i) and (iv) show the total photon number whereas (ii) shows the contribution from the environment and (iii) from the scattering. The parameters are $\Gamma_\phi=0$, $\nu=0.5\gamma$, and $\Omega=4\gamma$ in curves (i)–(iii), and $\Gamma_\phi=0.5\gamma$ and $\Omega=10\gamma$ in (iv). Further, $\eta=3\times10^{-3}$, and $Q=10^6$. Dashed lines show analytical fits to the numerical data using Eq. (10). Note that the $0\le\Gamma_a/\gamma\le1$ covers the whole range of all possible values of the subradiant decay rate.

According to Eq. (10), both the environmental and the scattering contributions to the cooling limit are suppressed by the reduced decay rate Γ_a of the subradiant state. This result is confirmed by our numerical results shown in Fig. 4. Interestingly, an optimal cooling limit is obtained for $\Gamma_a\approx0.05\gamma$. In this point, the total final phonon number is only 0.03, which corresponds to 97% occupation of ground state. Further decrease in Γ_a leads to an unexpected rapid increase in the cooling limit. The origin of this increase is that due to level shifts, the qubit transition frequency is slightly off-resonant with the driving field frequency. If the scattering state becomes too narrow, then the driving field cannot excite the cooling transitions any more. We verified this interpretation by a suitable slight change in the driving field frequency, which allows to achieve effective cooling also for smaller Γ_a. With additional pure dephasing $\Gamma_\phi=0.5\gamma$, the optimum cooling limit is obtained for $\Omega=10\gamma$. For $\Gamma_a=0.05\gamma$, the steady state is $n_{ss}\approx0.29$, which indicates 78% occupation of ground state. In experiments, inaccuracies of the positioning of the qubits can lead to an asymmetric coupling which can change the decay rate Γ_a. But the transition frequency of qubits we study is on the order of gigahertz corresponding to a wavelength of cm order. Currently, a superconducting qubit can be positioned with accuracy of micron. Thus the change in Γ_a resulting from the position derivation of qubit is small. It is also possible to fabricate qubits with uniform coupling to a driving line using existing techniques.[27,36] Thus, the asymmetry of the coupling can be made small, and thus has little effect on the cooling, since the NANMR can be cooled to its ground state over a wide range of subradiant damping rates Γ_a, see Fig. 4.

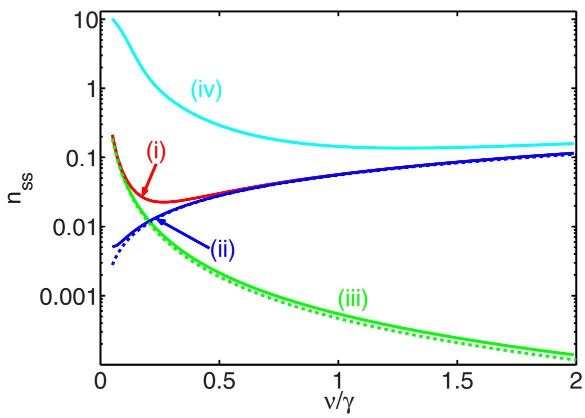

FIG. 5. (Color online) The final resonator phonon number n_{ss} against the resonator frequency ν. Curves and parameters as in Fig. 4, with $\Gamma_a=0.05\gamma$. Note that $0\leq\Gamma_a/\gamma\leq 1$ covers the whole range of possible values for Γ_a.

Next, we study the dependence of the cooling limit on the resonance frequency ν of the NAMR, see Fig. 5. The environmental contribution to n_{ss} is proportional to ν. Our scheme thus provides particularly efficient cooling of NAMRs with low frequencies. But since the scattering contribution is inversely proportional to the square of ν, this part becomes eventually dominant with decreasing ν. We find an optimal point of $\nu\approx 0.25\gamma$ where a minimum final phonon number of 0.02 is achieved. This number agrees well with the optimal value $\nu_{opt}=\sqrt[3]{\eta^2\Omega^2 Q\Gamma_a G/8CN_i}$ evaluated from Eq. (10) with $C=0.4$ and $G\sim 3$. When including a pure dephasing of $\Gamma_\phi=0.5\gamma$, an optimal cooling limit of $n_{ss}=0.15$ is obtained for $\Omega=10\gamma$ and $\nu=\gamma$.

IV. SUMMARY

In summary, we have presented an efficient ground-state cooling scheme for a NAMR by coupling it to two interacting flux qubits. We found that collectivity enhances the cooling performance in two crucial ways. First, the collective frequency shift of the qubit states effectively eliminates unwanted carrier excitations. Second, cooling via the subradiant Dicke state suppresses both contributions from the thermal environment and the scattering of the cooling field to the cooling limit. As a result, ground-state cooling can be achieved even in the nonresolved regime, and using rather small driving fields.

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